TWISTING (TORSIONAL) STIFFNESS

Barry T. Cease

Cease Industrial Consulting

ceasevibration@icloud.com

843-200-9705

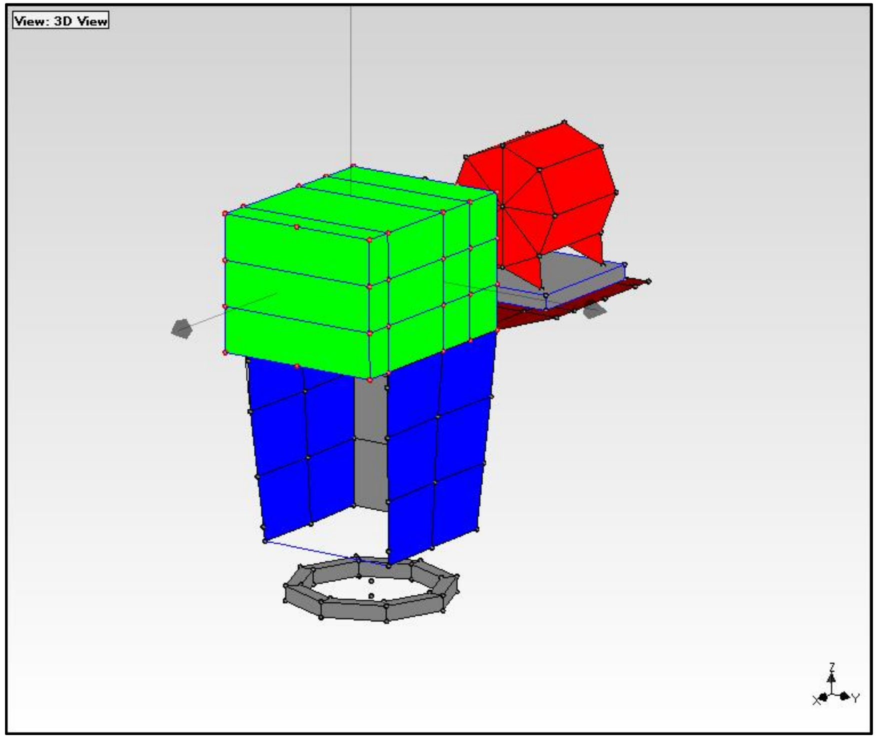
USUAL STIFFENING PRACTICES

- Most vibration analysts are familiar with methods of how to increase or decrease machine or structural stiffness in the horizontal, vertical, or axial directions.
- Typical methods include adding braces, gussets, etc to increase a machine's stiffness in the direction and at the location(s) where vibration is excessive.
- Many times these stiffness changes are performed to modify a machine or structure's natural frequency in an attempt to avoid a destructive resonance problem.
- Many of these stiffening techniques are self-evident when performed to reduce vibration in the radial or axial directions, but what do we do when our machine or structure's vibration isn't in any of these directions but instead is in the twisting or torsional direction?

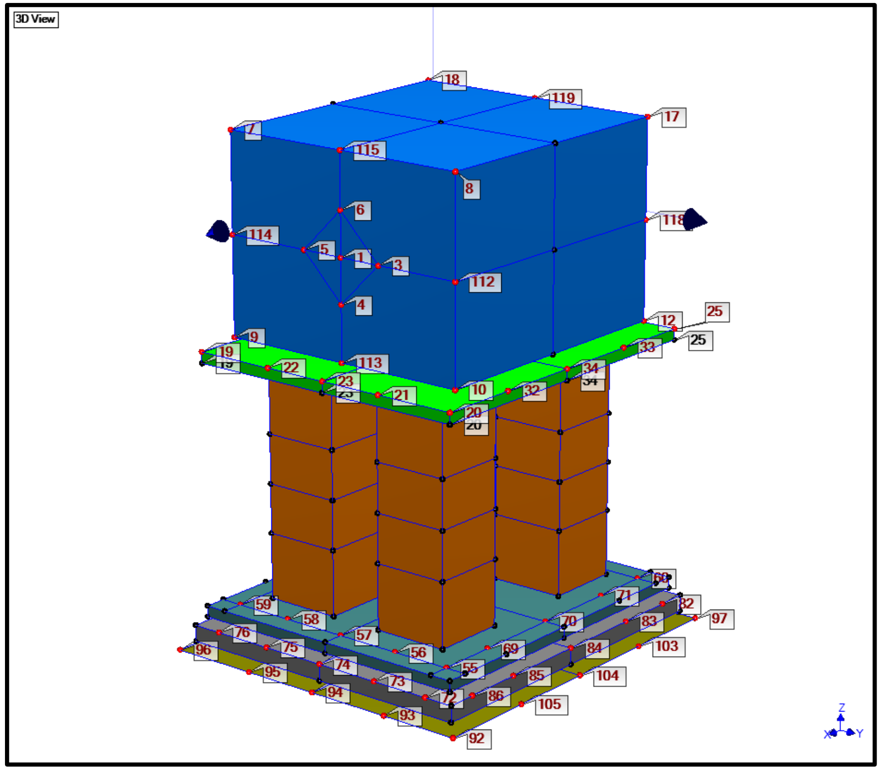
TORSIONAL OR TWISTING VIBRATION

- So what do we mean when we speak of torsional or twisting vibration?
- Torsional shaft vibration is not the topic of this discussion. We are discussing torsional or twisting vibration of the entire machine, base, or structure.
- It essentially means vibration seen as rotation about an axis along a machine or structure.
- How do we go about stiffening against this sort of twisting vibration?

TWO EXAMPLES OF TWISTING VIBRATION



Vertical Agitator
Twisting vibration occurring at 1x rpm motor (1,790 cpm) about the Z axis (vertical).



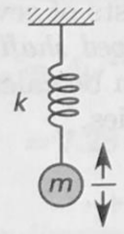
Roll Drive Motor
Twisting vibration occurring at 2x rpm motor (~ 3,200 cpm) about the Z axis (vertical).

TORSIONAL SPRING CONSTANT (STIFFNESS)

- Just like the linear springs most of us are familiar with, there are torsional springs we use in things like clocks, clothes pins, vehicle suspensions, door hinges, and many other applications.
- Increasing a structure’s torsional stiffness will usually involve increasing a quality known as its polar moment of inertia or J.
- The polar moment of inertia (J) is often thought of as a measure of a structure’s or object’s resistance to twisting.
- Object’s or structure’s with high polar moment of inertia will have high resistance to twisting and high torsional stiffness.

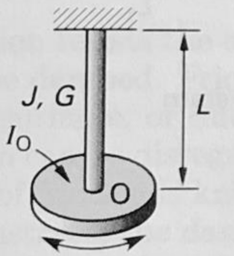
LINEAR & TORSIONAL VIBRATING SYSTEMS

LINEAR SPRING

mechanism	natural frequency (ω)	linear frequency (f)	period (T)
 mass and spring	$\sqrt{\frac{k}{m}}$	$\left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$	$2\pi\sqrt{\frac{m}{k}}$

Formulas for the natural frequency & period of a simple, single degree of freedom, linear vibrating system. Linear stiffness "k".

TORSIONAL SPRING

mechanism	natural frequency (ω)	linear frequency (f)	period (T)
 torsional mass and spring	$\sqrt{\frac{JG}{I_0 L}}$	$\left(\frac{1}{2\pi}\right)\sqrt{\frac{JG}{I_0 L}}$	$2\pi\sqrt{\frac{I_0 L}{JG}}$

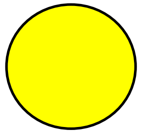
Formulas for the natural frequency & period of a simple, single degree of freedom, torsional vibrating system (torsional pendulum). Torsional stiffness G*J/L.

Shear Modulus ("Material Stiffness")

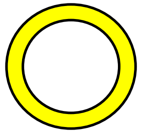
Polar Moment Of Inertia ("Geometric Stiffness")

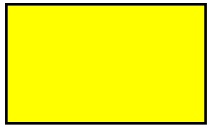
<u>Source</u>: "Mechanical Engineering Reference Manual", Lindeburg, Michael R., 1998, Professional Publications

FORMULAS FOR POLAR MOMENT OF INTERTIA (J)

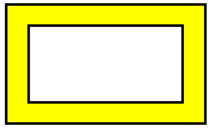
- Circular Shaft or Beam  J = $\pi d^4/_{32}$

  So as the diameter (d) of the beam increases, its polar moment of inertia increases as does its torsional stiffness.

- Hollow Circular Tube  J = $\frac{\pi}{32}(d_o{}^4 - d_i{}^4)$

- Rectangular Beam  J = $\frac{bh}{12}(h^2 + b^2)$

  So as either the base (b) or the height (h) of the beam increases, its polar moment of inertia increases as does its torsional stiffness.

- Hollow Rectangular Tube  J = $J_o - J_i = \frac{b_o h_o}{12}(h_o{}^2 + b_o{}^2) - \frac{b_i h_i}{12}(h_i{}^2 + b_i{}^2)$

The common conclusion from all four of these shapes is that in general the bigger their diameter or size the higher their torsional stiffness will be. In the case of the hollow beams, it is also true that the greater the thickness, the higher the torsional stiffness, but to a lesser degree.

POLAR MOMENT OF INERTIA COMPARISON (SIMILARLY SIZED SHAPES)

- A solid circular beam of 8” diameter: Polar Moment Of Inertia (centroid), J = 402 in^4.
- A hollow circular tube of 8” outer diameter and 6” inner diameter (1” wall): J = 275 in^4
- A hollow circular tube of 8” outer diameter and 7” inner diameter (0.5” wall): J = 166 in^4
- An 8” solid square beam: J = 683 in^4
- A square tube of 8” outer and 6” inner dimensions (1” wall): J = 467 in^4
- A square tube of 8” outer and 7” inner dimensions (0.5” wall): J = 283 in^4
- A channel with an 8” web, 8” flange & 1” wall: J = 375 in^4
- An 8” x 8” angle with 1” wall: J = 258 in^4
- An I-beam (W) with a 8” web, 8” flange & 1” wall: J = 385 in^4

So the morale of this story is that geometry matters. The geometry of both the solid and hollow square beams offer very high torsional stiffness for their size. The solid and hollow circular beams offer good torsional stiffness as well. The worst geometries for torsional stiffness were the channel, angle, and I-beams.

RULES OF THUMB

Good “rules of thumb” when looking for ways to effectively stiffen a machine or structure:

1) Work with the OEM if at all possible. A solution to your problem may already be known and implemented somewhere else. They also might have resources like structural engineers that can make solving your problem much easier.
2) Try to connect or attach points of high deflection to points of low deflection.
3) Make sure the new “attachments” (bracing, gussets, etc) offer above average strength in the direction the vibration is dominant (ie: if you’re dealing with horizontal vibration, ensure that any bracing or gussets are attached either in the horizontal direction or as close to it as you can get).
4) For torsional or twisting vibration problems, try moving towards a square or circular structure if possible. Eliminate any voids that might exist along the structure or install bolted metal doors instead of plastic ones. If possible connect discrete columns together into a much larger continuous support. If possible, shorten the structure’s height. Avoid channels or I-beam structures. If you have channel or I-beam structures, try “boxing them in” by welding plate between the flanges. Beware of the torsional “weak link” along a structure or machine (voids, couplings, etc); this weak point will dictate the torsional stiffness of the whole.